was likely being driven by the canister turntable frequency and its first multiple. Although there is clear qualitative agreement between theory and experiment, the theoretical resonance occurred at a longer boom length than the experiment (165 vs 92 in.). It is believed that this difference is due to the representation of a complex joint-dominated truss structure by a Timoshenko beam with uniform properties and perfect boundary conditions. Although a modification of the properties of the beam could be attempted to more accurately match the experimental results, such an effort was not performed in this study.

Summary

The deployment of a flexible boom from an oscillating base may result in potentially large resonant-driven bending moments that can be detrimental to the boom structure. In systems such as the one studied here, the base oscillation, due to interaction of a rotating turntable device with slight freeplay within the deployment canister, should be characterized experimentally prior to flight to ensure boom health.

Appendix: Matrix Definitions

If z is defined as $z^T = \{q_1 q_2 \ldots, s_1 s_2 \ldots\} = \{z_1, z_2\}$, the definitions of the nonzero elements of the matrices appearing in the transverse equations of motion are given as the following:

$$M_{11ij} = \int_0^L \rho \psi_i \psi_j \, dx + m_T \psi_{iL} \psi_{jL}$$

$$M_{22ij} = \int_0^L I_z \phi_i \phi_j \, dx + I_{zT} \phi_{iL} \phi_{jL}$$

$$B_{11ij} = \int_0^L \rho \dot{\psi}_i \psi_j \, dx + m_T \dot{\psi}_{iL} \psi_{jL}$$

$$B_{22ij} = \int_0^L I_z \dot{\phi}_i \phi_j \, dx + I_{zT} \dot{\phi}_{iL} \phi_{jL}$$

$$D_{11ij} = \int_0^L \rho \dot{\psi}_i \dot{\psi}_j \, dx + m_T \dot{\psi}_{iL} \dot{\psi}_{jL}$$

$$D_{22ij} = \int_0^L I_z \dot{\phi}_i \dot{\phi}_j \, dx + I_{zT} \dot{\phi}_{iL} \dot{\phi}_{jL}$$

$$f_{1i} = 2\int_0^L \rho\left[-\dot{L}\dot{\gamma}\psi_i + (\dot{\delta} + \dot{\gamma}x)\dot{\psi}_i\right] dx + 2m_T\left[-\dot{L}\dot{\gamma}\psi_{iL} + (\dot{\delta} + \dot{\gamma}L)\dot{\psi}_{iL}\right]$$

$$f_{2i} = 2\int_0^L I_z \dot{\gamma}\dot{\phi}_i \, dx + 2I_{zT}\dot{\gamma}\dot{\phi}_{iL}$$

$$g_{1i} = 2\int_0^L \rho(\dot{\delta} + \dot{\gamma}x)\psi_i \, dx + 2m_T(\dot{\delta} + \dot{\gamma}L)\psi_{iL}$$

$$g_{2i} = 2\int_0^L I_z \dot{\gamma}\phi_i \, dx + 2I_{zT}\dot{\gamma}\phi_{iL}$$

$$K_{11ij} = \int_0^L \kappa GA \left(\frac{\partial \psi_i}{\partial x}\right)\left(\frac{\partial \psi_j}{\partial x}\right) dx$$

$$K_{12ij} = -\int_0^L \kappa GA \left(\frac{\partial \psi_i}{\partial x}\right)\phi_j \, dx = K_{21ji}$$

$$K_{22ij} = \int_0^L \left[EI\left(\frac{\partial \phi_i}{\partial x}\right)\left(\frac{\partial \phi_j}{\partial x}\right) + \kappa GA \, \phi_i \phi_j\right] dx$$

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Eigenstructure Assignment for the Extended Medium Range Air-to-Air Missile

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Introduction

EIGENSTRUCTURE assignment is applied to the design of an autopilot for the extended medium range air-to-air technology (EMRAAT) missile. We compute eigenstructure assignment feedback gains by choosing desired eigenvectors based on mode decoupling and using the orthogonal projection solution suggested by Andry et al.[1] to compute achievable eigenvectors. We compare our solution to the linear quadratic regulator design that was proposed by Bossi and Langehough.[2] An important difference between this application and other eigenstructure assignment applications that have appeared in the literature is that the lateral dynamics of the EMRAAT missile does not have a well-defined Dutch roll mode. Therefore, eigenstructure assignment is utilized not only for mode decoupling, but also to create distinctly separate Dutch roll and roll modes.

Eigenstructure Assignment

Consider a linear time-invariant multi-input multi-output system described by the triple (A, B, C). The eigenstructure assignment problem was considered by Andry et al.[1] who have shown the need for the eigenvector v_i to be in the subspace spanned by the columns of $(\lambda_i I - A)^{-1}B$. In general, however, a desired eigenvector v_i^d will not reside in the prescribed subspace and, hence, cannot be achieved. Instead, it is suggested in Ref. 1 that an achievable eigenvector be chosen as the projection of v_i^d onto the subspace that is spanned by the columns of $(\lambda_i I - A)^{-1}B$.

Missile Autopilot Design

Consider the EMRAAT bank-to-turn missile, which is described by Bossi and Langehough.[2] A sixth-order model of the yaw/roll dynamics at a 10-deg angle of attack is con-

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Table 1 Comparison of extended medium range air-to-air technology designs

Closed-loop eigenvalues	Feedback gain matrix
Open loop	
$\lambda_{1,2}=-1.31\pm j24.0$ $\lambda_3=-0.549$ $\lambda_4=0.0$ $\lambda_{act}=-179.0$ $\lambda_{act}=-179.0$	N/A
Linear quadratic regulator	
$\lambda_{1,2}=-34.77\pm j16.5$ $\lambda_3=-5.11$ $\lambda_4=-16.35$ $\lambda_{act}=-111.8$ $\lambda_{act}=-158.4$	$\begin{array}{c} \beta \quad r \quad p \quad p_I \\ \begin{bmatrix} -1.83 & 0.154 & -0.00499 & 0.0777 \\ 2.35 & -0.288 & 0.0358 & -0.0201 \end{bmatrix} \begin{matrix} \delta_r \\ \delta_a \end{matrix} \end{array}$
Eigenstructure assignment design	
$\lambda_{Dr}=-23.98\pm j17.99$ $\lambda_{roll}=-10.01\pm j9.98$ $\lambda_{act}=-132.1$ $\lambda_{act}=-161.1$	$\begin{array}{c} \beta \quad r \quad p \quad p_I \\ \begin{bmatrix} -4.19 & 0.233 & 0.00374 & 0.731 \\ 2.89 & -0.290 & 0.00631 & -0.812 \end{bmatrix} \begin{matrix} \delta_r \\ \delta_a \end{matrix} \end{array}$

Table 2 Eigenvectors for the extended medium range air-to-air technology designs[a]

Open loop

	Mode 1	Mode 2	Mode 3		
β	0.0	0.000254	−0.000244	$\pm j$	−0.00744
r	0.0	−0.0968	−0.00522		−0.000182
p	0.0	−0.5490	<u>1.0</u>		0.0
p_I	<u>1.0</u>	<u>1.0</u>	−0.00227		−0.0415
δ_r	0.0	0.0	0.0		0.0
δ_a	0.0	0.0	0.0		0.0

Linear quadratic regulator

	Mode 1	Mode 2	Mode 3		
β	0.00212	0.00182	−0.00589	$\pm j$	−0.00274
r	0.187	0.208	−0.07506		0.00399
p	<u>1.0</u>	<u>1.0</u>	<u>1.0</u>		0.0
p_I	−0.196	−0.0612	−0.0235		−0.0111
δ_r	0.00483	0.0208	−0.00865		0.00690
δ_a	−0.00933	−0.0204	0.0529		−0.0152

Eigenstructure assignment design

	Roll mode			Dutch roll mode		
β	−0.00867	$\pm j$	−0.00912	0.0259	$\pm j$	0.0201
r	0.000757		0.0000898	<u>1.0</u>		0.0
p	<u>1.0</u>		0.0	0.0190		0.0261
p_I	−0.0501		−0.0499	0.0000142		−0.00108
δ_r	0.00393		0.00158	0.131		−0.113
δ_a	0.0238		0.00136	−0.237		0.0958

[a]Actuator mode eigenvectors are not shown.

sidered with state vector, control vector, and measurement vector given by $x=[\beta,r,p,p_I,\delta_r,\delta_a]^T$, $u=[\delta_r,\delta_a]^T$, and $y=[\beta,r,p,p_I]^T$, respectively. Here, β is the sideslip angle (deg), r the yaw rate (deg/s), p the roll rate (deg/s), p_I the integrated roll rate (deg), δ_r the rudder deflection (deg), and δ_a the aileron deflection (deg). The state space matrices A, B, and C are shown in Ref. 2. The design described in Ref. 2 uses a full state feedback linear quadratic regulator design for the fourth-order reduced-order model, which does not include the actuator dynamics. The optimal eigenvalues for the fourth-order model are $\lambda_{1,2}=-24\pm j18$, $\lambda_3=-5.1$, and $\lambda_4-14.5$. The closed-loop eigenvalues for the sixth-order model are shown in Table 1 where we observe that the eigenvalues are no longer at their optimal locations. The open- and closed-loop eigenvectors for the dominant modes of the sixth-order model are shown in Table 2. We observe that the open-loop eigenvectors each have their largest entry in either roll rate or integrated roll rate. Thus, there is no well-defined Dutch roll mode (composed predominantly of yaw rate and sideslip angle), which is quite different from most other aircraft and missile control designs that have appeared in the literature. We observe that the linear quadratic regulator design has caused the largest entry in each eigenvector to correspond to roll rate. Hence, a small perturbation in sideslip angle will cause large roll rate motion for both the open-loop and the linear quadratic regulator design. We note that the design of Ref. 2 yields a closed-loop system that does not exhibit a clearly defined Dutch roll mode. The sideslip and roll rate responses for the closed-loop missile for a 1-deg initial sideslip angle are shown in Figs. 1 and 2, respectively. We observed that the Ref. 2 design exhibits acceptable overshoot and settling time, but the sideslip angle induces an unacceptably large roll rate response with a peak value of −50.1 deg/s.

Next, we design an eigenstructure assignment autopilot by using the same four measurements that were used in the Ref. 2 design. The desired eigenvalues and desired eigenvectors are shown below where we can specify four closed-loop eigenvalues because we have four measurements

Dutch roll mode:

$$v_{dr}^d=\begin{bmatrix}1\\x\\0\\0\\x\\x\end{bmatrix}\pm j\begin{bmatrix}x\\1\\0\\0\\x\\x\end{bmatrix}$$

$$\lambda_{dr}^d=-24\pm j18$$

Roll mode:

$$v_{\text{roll}}^d=\begin{bmatrix}0\\0\\1\\x\\x\\x\end{bmatrix}\pm j\begin{bmatrix}0\\0\\x\\1\\x\\x\end{bmatrix}$$

$$\lambda_{\text{roll}}^d=-10\pm j10$$

We remark that the desired eigenvectors are chosen to create a Dutch roll mode that is predominantly yaw rate and sideslip angle together with a roll mode that is predominantly roll rate

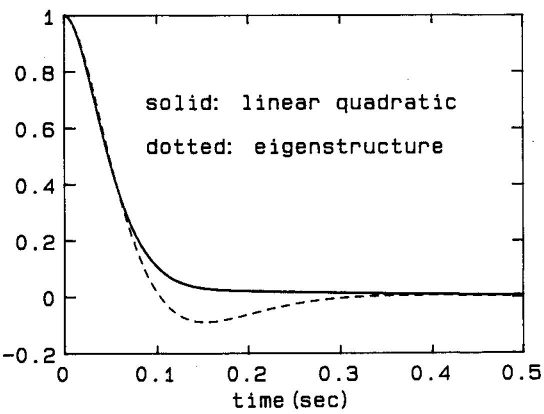

Fig. 1 Sideslip responses to 1-deg initial sideslip.

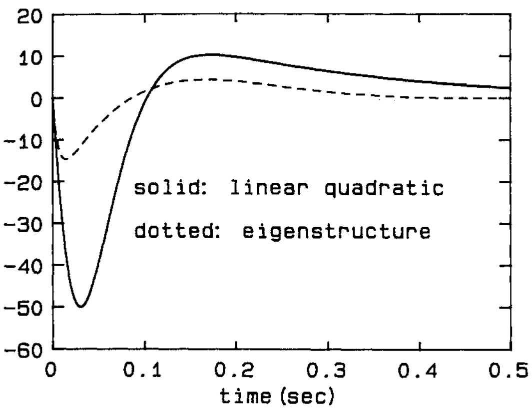

Fig. 2 Roll rate responses to 1-deg initial sideslip.

and integrated roll rate. The zero eigenvector entries are chosen so as to decouple the Dutch roll mode and the roll mode. The achievable eigenvectors are shown in Table 2 where we observe that we have obtained clearly defined Dutch roll and roll modes. The roll mode eigenvector is characterized by its largest entry corresponding to roll rate with very small entries corresponding to sideslip angle and yaw rate. The Dutch roll eigenvector is characterized by its largest entry corresponding to yaw rate with a very small entry corresponding to integrated roll rate and a small entry corresponding to roll rate. We would desire that the entry corresponding to roll rate be even smaller. Nevertheless, this new eigenstructure design should exhibit significantly improved decoupling between sideslip angle and roll rate. The sideslip and roll rate responses for the eigenstructure assignment autopilot to a 1-deg initial sideslip are shown in Figs. 1 and 2, respectively. We observe that a significant improvement is obtained in the decoupling between sideslip angle and roll rate. The peak value of roll rate is now -14.2 deg/s, which compares with -50.1 deg/s for the linear quadratic regulator design and represents an improvement of approximately 72%.

The maximum aileron deflections, aileron deflection rates, rudder deflections, and rudder deflection rates are 1.68 deg, 175.8 deg/s, 1.31 deg, and 119.6 deg/s, respectively, for the linear quadratic design and 1.90 deg, 211.0 deg/s, 3.16 deg, and 263.7 deg/s, respectively, for the eigenstructure assignment design. These maximum deflection rates are within the expected 400-deg/s limit for the advanced state-of-the-art electromechanical actuator described by Langehough and Simons.[3] The condition numbers of the modal matrices, which are a commonly used measure of eigenvalue sensitivity, are 200.77, 766.03, and 115.77 for the open-loop design, linear quadratic design, and eigenstructure assignment design, respectively. We observe that the linear quadratic regulator design exhibits a modal matrix condition number that is 3.8 times larger than the open-loop value, whereas the eigenstructure assignment design has a condition number that is almost half the open-loop value. Thus, the eigenstructure assignment design exhibits improved decoupling between sideslip and roll rate together with improved eigenvalue sensitivity, albeit at the expense of larger actuator deflection and deflection rates.

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Eigensystem Assignment with Output Feedback

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Introduction

EIGENVALUE and eigenvector assignment techniques have been extensively used in the active control design of linear, time-invariant systems. Numerous methods and algorithms involving both constant full-state feedback and output feedback have been developed.[1-14] Within the past decade, several iterative and noniterative algorithms[9-14] have been derived to exploit the freedom offered beyond the eigenvalue assignment by multi-inputs and multi-outputs, to either improve the performance of the closed-loop system or minimize the required control effort. Among the class of iterative methods is Kautsky's algorithm[9] that iteratively minimizes some robustness measure of the closed-loop system in terms of the conditioning of the closed-loop modal matrix through an orthogonal projection approach. Direct nonlinear programming techniques were used to minimize scalar robustness measures such as closed-loop conditioning[10,11] and closed-loop normality indices.[13] In the class of noniterative methods, a recent algorithm by Juang et al.[12] can be identified wherein closed-loop eigenvectors are chosen as such to maximize their orthogonal projection to the open-loop eigenvector matrix or its closest unitary matrix, thereby maximizing the robustness of the closed-loop system. A sequential algorithm by Maghami and Juang[14] can also be identified that utilizes Schur decomposition and Givens rotations to assign the desired closed-loop eigenvalues via full-state or output feedback. Many of the existing methods require full-state feedback that is not practical in lieu of the recent trends toward the erection and deployment of large flexible structures having thousands of degrees of freedom. Even those methods that can implement output feedback designs either do not take advantage of the full freedom of the system and/or are not computationally feasible.

In this paper, a new approach for the eigenvalue and eigenvector assignment of linear first-order, time-invariant systems is developed. The approach extends the procedure outlined in Ref. 12 by allowing the assignment of the maximum possible number of closed-loop eigenvalues via constant output feedback. The system is assumed to be fully controllable and observable, having full rank input and output influence matrices. The approach starts with the generation of a collection of bases for the space of attainable closed-loop eigenvectors corresponding to the desired closed-loop eigenvalues. The singular value decomposition (SVD) or QR decomposition are used

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